RESIDUE GENERATION, THE STUDY OF THEIR PROPERTIES

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Abstract: in this paper is presented the comparison between the local residues and the global residues and their properties. It is also emphasized the wish of producing local and global models as smaller subsystems, comparing the localizing properties obtained, with the properties of the local residues

1. Local residue generation

The local models that have been obtained describe a part of the studied system. They do not yet represent diagnosis models because they do not allow the evaluation of the coherence between the action they describe and the available observations. The objective of the residue generator is to evaluate these coherences. The principle is: the local model M_{Vi} is used in order to calculate V_i and $\hat{V}_i$. Thus the local residue r_{Vi} generation is made possible, using the difference between V_i and $\hat{V}_i$. Thus, the local residue corresponds neither more nor less to the output error of the local model associated to it. Taking into account the algorithm of producing a local model M_{Vi}, its input variables are either exogenous, $L^{Vi}_{Vex} \subset L_{Vex}$, or measurement variables, $L^{Vi}_{Vm} \subset L_{Vm}$.

$$V_i = M_{Vi}\left(L^{Vi}{}_{Vm}, L^{Vi}{}_{Vex}\right) \tag{1}$$

The unit of the variables studied in the application is marked with L_{Vobs}, $L_{Vnonobs}=L_V \backslash L_{Vobs}$ represents the unit of the unobserved variables. All measurement variables are observed variables ($L_V \subset L_{Vobs}$) but this is not a necessary condition in the case of the exogenous variables: a certain value corresponds to a known exogenous variable but the perturbation or the effect can be modeled by means of an unknown exogenous input. The prediction of V_i cannot be calculated but by means of the available observations; the unknown variables must be null in order to make the simulation possible.

$$\hat{V}_i = M_{Vi}\left(L^{Vi}{}_{Vm}, L^{Vi}{}_{Vex} \cap L_{Vobs}, \left(L^{Vi}{}_{Vex} \cap L_{Vnonobs}\right) = 0\right) \tag{2}$$

$$\hat{r}_{Vi} = V_i - \hat{V}_i \tag{3}$$

2. The properties of the local residues.

Referring to (1) and (2) in the relation (3) we have a new expression of $\hat{r}_{Vi}$.

$$\begin{aligned}\hat{r}_i = {} & M_{Vi}\left(L^{Vi}vm, L^{Vi}vex \cap L_{Vobs}, L^{Vi}vex \cap L_{Vnonobs}\right)\ldots \\ & - M_{Vi}\left(L^{Vi}vm, L^{Vi}vex \cap L_{Vobs}, \left(L^{Vi}vex \cap L_{Vnonobs}\right) = 0\right)\end{aligned} \tag{4}$$

Assuming that the equations perfectly model the system[1], the relation 4 shows that the only variables that can determine $\hat{r}_{Vi}$ to be not null are the unobserved exogenous variables which intervene in the local model $M_{Vi}(L^{Vi}_{Vex} \cap L_{Vnonobs})$. In order to be sure that a variable $V \in L^{Vi}_{Vex} \cap L_{Vnonobs}$ affects $\hat{r}_{Vi}$ while it becomes not null, it is necessary to check if there is a connection between V and V_i. Such a connection exists only due to the producing algorithm of the local models. We would say that $\hat{r}_{V_i}$

[1] If there is not necessary, we can't assume that M_{Vi} in relation (1) is strictly identical with M_{Vi} in relation (2).

is causally sensitive to V. This wording tends to make distinction between the existence of a path, information about the causal structure, and sensibility, in the classic sense of the word. For instance, the residue r=U·V is causally sensitive to V (there is a connection between V and r, in other words, a variation of V can be propagated on r); although the sensibility of r to V can be null if U is annulled. Different from the concept of "sensitive" residue, which requires that the equation content to be known, the concept of "causally sensitive" residue doesn't require but the causal and the structural information. These definitions being given, we can enunciate the localization properties of the local residues (table 1).

Table 1 The localization properties of the local residues

The localization properties of the local residues: Assuming that the equations perfectly model the system, the local residue $\hat{r}_{V_i}$ associated to a local model M_{Vi} is causally sensitive exclusively to the unobserved exogenous variables that intervene in $M_{Vi}(L_{Vex}^{Vi} \cap L_{Vnonobs})$. Assuming that there are modeling errors, only those errors that affect the equations L_E^{Vi} appearing in M_{Vi} can make that $\hat{r}_{V_i}$ be null.

It is also emphasized the wish of producing local models as smaller subsystems for which the output represents a measurement variable and the inputs are exogenous or measurement variables: thus, the breaking-up of the global model is thorough, in other words, the local models are multiple and of small size and they have better localization properties.

3. Global residues

A second set of residues may be calculated for each measurement variable: the global residue $\hat{r}_{V_i}$ associated to the measurement variable $V_i \in L_{Vm}$ corresponds to the output error of the global model M of the studied system.

$$[\cdots\ V_i\ \cdots]^T = M(L_{Vex}) \quad (5)$$

$$[\cdots\ V_i^*\ \cdots]^T = M(L_{Vex} \cap L_{Vobs}, (L_{Vex} \cap L_{Vnonobs}) = 0) \quad (6)$$

$$r^* = [\cdots\ r_{Vi}^*\ \cdots]^T = [\cdots\ V_i\ \cdots]^T - [\cdots\ V_i^*\ \cdots]^T \quad (7)$$

The relation (5) shows that the global model combines all the exogenous variables with all the measurement variables. The relation (6) shows how can the reference value of V_i, V_i^* be calculated by means of the observable exogenous variables; just like in relation 18 the exogenous variables (that model the perturbations, the faults etc.) are null in order to make the simulation possible. Consequently, V_i^* may be considered a homogenous reference of V_i due to the faultless and perturbationless action of the global system. (as well as in relation (18)), $\hat{V}_i$ may be considered a homogenous reference of V_i due to the faultless and perturbationless action of the local system).

4. The properties of the global residues

The same reasoning like in relation 19. is applied to the global residues, referring to the relations 5 and 6 in the relation 7 and there is obtained a new expression of r^*:

$$r^* = [\cdots \ r_{Vi}^* \ \cdots]^T = M(L_{Vm} \cap L_{Vobs}, L_{Vex} \cap L_{Vnonobs})\ldots$$
$$- M(L_{Vex} \cap L_{Vobs}, (L_{Vex} \cap L_{Vnonobs}) = 0) \quad (8)$$

Assuming that the equations perfectly model the system, the relation (8) shows that the only variables that can determine the global residues to be not null are unobserved exogenous variables ($L_{Vex} \bigcap L_{Vnonobs}$). In order to be sure that a variation of $V \in L_{Vex} \bigcap L_{Vnonobs}$ can be propagated on r_{Vi}^*, or, in other words, in order to check if r_{Vi}^* is causally sensitive to *V*, we must check if there is a connection between *V* and V_i in the causal graph of the system. The localization properties of the global residues are presented in table 2.

Table 2 The localization properties of the global residues

The localization properties of the global residues: Assuming that the equations perfectly model the system, the global residue r_{Vi}^* cannot be causally sensitive but to the unobserved exogenous variables r_{Vi}^* if there exists a connection between *V* and V_i in G^c. Assuming that there are modeling errors, all those errors which affect the equations appearing while calculating V_i by means of the exogenous outputs can make r_{Vi}^* to be not null.

5. Comparison between the properties of the global and the local residues.

Comparing the tables 1 and 2, we notice that the localization properties of the local residues are stronger than the properties of the global residues. This fact generates multiple interpretations:

1. In the case of the local residues, the problem of the paths between the unobserved exogenous variables and the model output is worked out by producing the local model itself.

2. As the local model is the smaller model possible, the set of the variables that could affect the associated local model is fundamentally smaller than a global residue.

3. The insertion of the measurements at a local model input (comparison of relations 2 and 6) means “the interruption” of the influence of some unobserved exogenous variables over the local residue. This is due to the fact that the measurements reflect the effects of such propagations.

A local analysis offers another vision over the localizing properties of the global residues, taking into account the item 3. A local model M_{Vi} being given we design in L_{Vm}^{Vi} the measurements variables at the input of $M_{Vi}(L_{Vm}^{Vi} \subset L_{Vm})$. Otherwise, the relation allows the calculation of a reference value for each measurement variable (L_{Vm}), hence, particularly, for each variable L_{Vm}^{Vi}. We have marked with $L^{Vi}{}_{V^*}$ the set of the reference values associated to the measurement variables L_{Vm}^{Vi} where the local expression of the global residue can be obtained as it follows:

$$V_i^* = M_{Vi}(L^{Vi}{}_{V^*}, L^{Vi}{}_{V_{ex}} \cap L_{Vobs}, (L^{Vi}{}_{V_{ex}} \cap L_{Vnonobs}) = 0) \quad (9)$$

$$r_{Vi}{}^* = V_i - V_i^* \quad (10)$$

Referring to the relations (1) and (9) in relation (10) we render evident the localizing properties of r_{Vi}^* from a local point of view.

$$r^*_{Vi} = M_{Vi}\left(L^{Vi}{}_{v_m}, L^{Vi}{}_{v_{ex}} \cap L_{Vobs}, L^{Vi}{}_{v_{ex}} \cap L_{Vnonobs}\right)\ldots$$
$$- M_{Vi}\left(L^{Vi}{}_{v^*}, L^{Vi}{}_{v_{ex}} \cap L_{Vobs}, \left(L^{Vi}{}_{v_{ex}} \cap L_{Vnonobs}\right) = 0\right) \quad (11)$$

The relation (11) shows that r^*_{Vi} can be affected by the unobserved exogenous variables which appear in M_{Vi}, on the one hand, and by all the possible deviation sources between the variables of L^{Vi}_{V*} si L^{Vi}_{Vm}, on the other hand; the application of the localizing properties of the global residues (Table 2) shows that these deviations correspond to a propagation (consequently to a path) between the unobserved exogenous variables and the M_{Vi} inputs. Comparing the localizing properties obtained like this, with the properties of the local residues (table 1) we may notice that the insertion of the measurements at the input of a local model means the "annihilation" of the influence on the local residue of some unobserved exogenous variables. Simultaneously, the insertion of the reference values at the input of the local models means the propagation of the deviations over the residue through those inputs.

CONCLUSIONS: one of the main interests of the local approach consists in the possibility of implementing an approach of the recursive localization: when a deviation at a variable is detected, in other words, when the respective global residue is not null, the testing of the local residue allows us to find out if the deviation origin is local or increasing[2]. In fact, if the local residue is almost zero it means that the subsystem described by the local model actions according to the proposed model; taking into account the existence of a deviation over the global residue we can conclude that the fault which causes this deviation has its origin in the superior part of the local subsystem. By a selective insertion of the measurements at the local model input (for instance, all measurements besides one), we can deduct through which input (inputs) of the local model the observed deviations take place [1]. A recursive reasoning that allows the recursive approach in order to localize the deviation does not justify but in the case of the systems which are very well equipped with control and measurement tools and which have a great number of variables.

6. Residue generation by composing local models.

For each measurement variable, a local model whose output corresponds to this variable has been realized. Actually, some inputs of the local models themselves are measurement variables. Consequently, it is possible the composition of more local models and the generation of new residue. The sensibility characteristics of these new residues are different from those of the local or global residues, and this contributes to the improvement of the localizing abilities. Let's consider M_{Vi} and M_{Vj} two local models so that $V_j \in L^{Vi}_{Vm}$

$$V_i = M_{Vi}\left(L^{Vi}{}_{Vm}, L^{Vi}{}_{Vex}\right) \quad (12)$$
$$V_j = M_{Vj}\left(L^{Vj}{}_{Vm}, L^{Vj}{}_{Vex}\right) \quad (13)$$

As $V_j \in L^{Vi}_{Vm}$, the conceiving of M_{Vi} and M_{Vj} is possible in order to form the model $M_{Vi(Vj)}$.

$$V_i = M_{Vi}\left(M_{Vj}\left(L^{Vj}{}_{Vm}, L^{Vj}{}_{Vex}\right), L^{Vi}{}_{Vm} - \{V_j\}, L^{Vi}{}_{Vex}\right) \quad (14)$$
$$V_i = M_{Vi(Vj)}\left(L^{Vj}{}_{Vm} \cup L^{Vi}{}_{Vm} - \{V_j\}, L^{Vj}{}_{Vex} \cup L^{Vi}{}_{Vex}\right) \quad (15)$$

[2] "Increasing" here, represents the orientation of the springs, deducted from the causal distribution. The oriented springs represent the possible propagation lines of the faults.

The compound model $M_{Vi(Vj)}$ has the same structure as the local model (compare the relations 15 and 12). The same reasoning as for the local models is applied in the relations 18 and 19, thus being possible the producing of the residue $\hat{r}_{Vi(Vj)}$ by means of this model. More than that, $\hat{r}_{Vi(Vj)}$ is causally sensitive to the unobserved exogenous variables which intervene in M_{Vi} or M_{Vj} $\left(L^{Vj}{}_{Vex} \cup L^{Vi}{}_{Vex}\right) \cap L_{Vnonpbs}$ and which does not reduce itself because of the combination of the two models. Such simplifications may appear at the level of the measurement equations that realize the junction between two local models. Figure 1 exemplifies this phenomenon (v is a measured variable, v_m is a measurement variable, Δv an exogenous variable that models a deviation converter and eq is the measurement equation of v).

CONCLUSIONS 1. In the absence of the simplifications related to the model producing, the new residues don't present more information than the initial local ones; consequently, there is no interest in conceiving the models.

CONCLUSIONS 2. Even if we haven't presented but the way of producing two local models, the process generalization for more than two models is evident. For instance, $M_{Vi(Vj,Vk(Vl))}$ represents the model resulted from the combination of M_{Vi} with M_{Vj} and M_{Vk}, the latter being itself compound with M_{Vl}; the residue calculated by means of $M_{Vi(Vj, Vk(Vl))}$ is $\hat{r}_{Vi\ (Vj,\ Vk(Vl)}$.

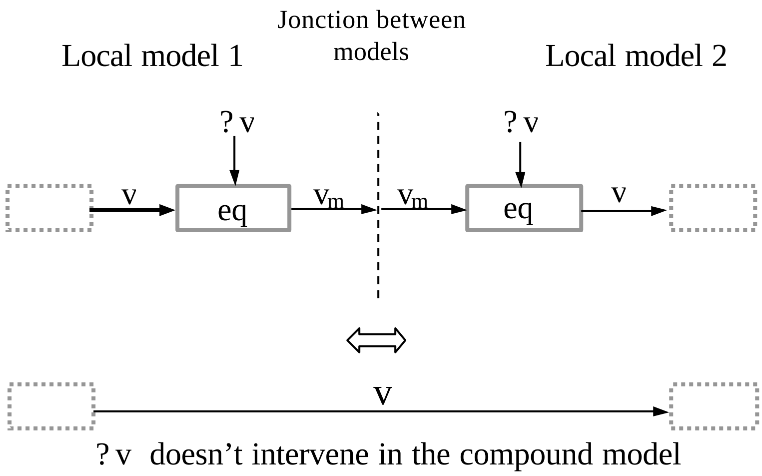

Figure 1 Simplification example related to the composition of two models

7. The application for DCM. Global and local residues

As all the measurement variables from the assembly ($L_{Vm} = \{\beta_m, I_m, \Omega_m\}$) are cardinal numbers three global residues will be realized, besides the residues resulted from the composition of the local models. The set of the observed and the unobserved variables is:

$$L_{Vobs} = \{I_{ref}, \beta_m, I_m, \Omega_m\} \quad (16)$$

$$L_{Vnonobs} = \{f_\beta, f_I, f_\Omega, f_U, d_\Gamma, X_I, \beta, I, \Omega, U, V\} \quad (17)$$

$$L_{Vex} \cap L_{Vnonobs} = \{f_\beta, f_I, f_\Omega, f_U, d_\Gamma\} \quad (18)$$

By means of the localizing properties of the local and global residues, we can establish tables of theoretical values that indicate the modeled fault to which each residue is sensitive.

Example 1: the global residue r_{Im}^* - by applying the properties from table 2 we can notice that r_{Im}^* can't be sensitive but to the unobserved exogenous variables. More than that, the causal graph of the DCM shows that there is a connection between f_β and I_m, between f_I and I_m , between f_U and I_m and between d_Γ and I_m. On the contrary, there is no connection between $f_?$ and I_m. Thus, we can draw the conclusion that r_{Im}^* is causally sensitive to f_β ,f_I, f_U, d_G and that r_{Im}^* is not sensitive to f_Ω. (Table 2.a)

Example 2: the local residue $\hat{r}_{Im}$: the local model associated to I_m is given by the relation We conclude that $L^{Im}{}_{Vm}$ ={β_m, Ω_m} and $L^{Im}{}_{Vex}$ { f_I, f_Ω ,f_U }. The application of the properties from the table 1 shows that $\hat{r}_{Im}$ is causally sensitive exclusively to variables f_I, f_Ω ,f_U (Table 2.b).

	$f_ß$	f_I	f_O	f_U	d_G
$r_{ßm}^*$	1	1	0	1	1
r_{Im}^*	1	1	0	1	1
r_{Om}^*	1	1	1	1	1

Global residues

	$f_ß$	f_I	f_O	f_U	d_G
$r_{ßm}^*$	1	0	0	0	0
r_{Im}^*	0	1	1	1	0
r_{Om}^*	0	1	1	0	1

Local residues

Table 2.. Tables of theoretical values (global and local residues)

Table 3 shows the importance of the local residues: they do not only allow the separation of $f_\beta, \{f_I, f_\Omega\}$, f_U from the influence of {$f_ß$, f_I, f_U} and f_O afferent to the local residues, two of them being perfectly decupled from the perturbation that is due to the resistant couple. Logically, the local residue that rests sensitive to the resistant couple is the one realized by means of the mechanic equation of the DCM.

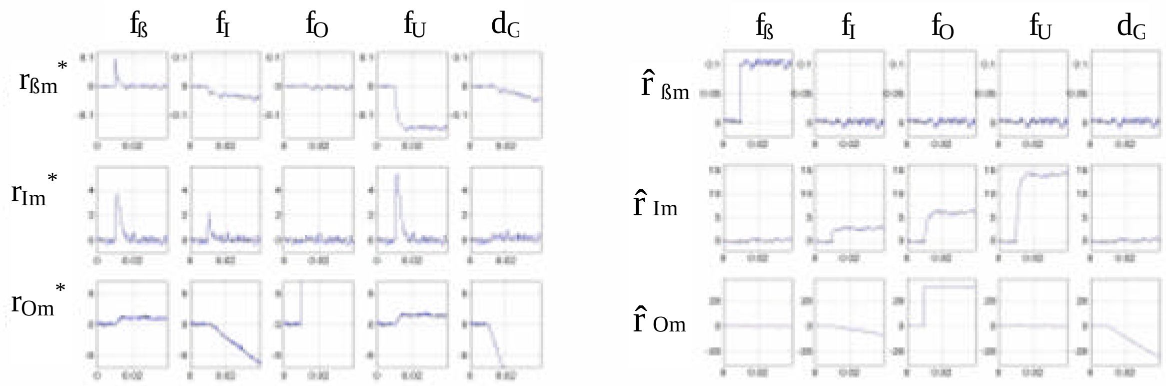

Figure 3. Checking and simulation of the tables of theoretical values

By means of figure 3 we can check through the simulation method the tables of theoretical values of the local and global residues for the lap defaults. The data basis relating to the equation content intervenes only in the stage of these simulations. Some residues seem to respond to some faults by a certain disturbance; in fact, it is the initial response that corresponds to the mechanical time coefficient of the DCM, which is slower than the electric time coefficient.

7.2. The residues associated to the local model composition

The process of the local models composition is now applied to the DCM. The set of the combinations in which a local model appears for one time is shown in figure 4. The theoretical values deducted from figure 4 are shown in table 4.

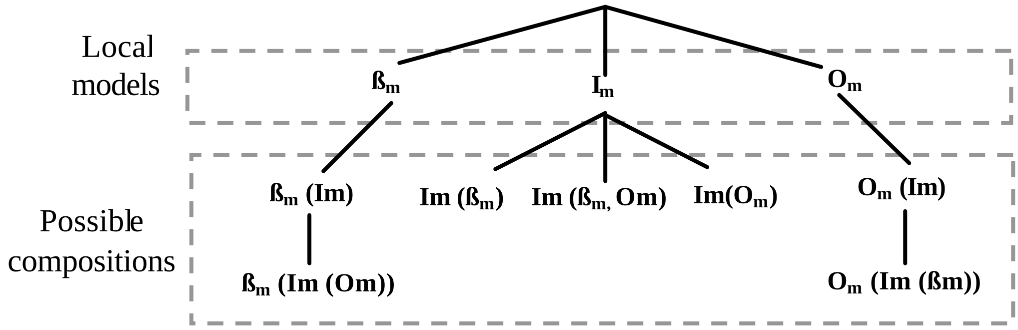

Figure 2.11 Possible compositions of the local models

Table 4. Table of theoretical values (composition of the local models)

	$f_ß$	f_I	f_O	f_U	d_G
$r_{Om\ (Im)}$	0	0	1	1	1
$r_{Im\ (Om)}$	0	1	0	1	1
$r_{ßm\ (Im)}$	1	1	1	1	0
$r\ I_{m\ (ßm)}$	1	1	1	1	0
$r_{ßm\ (Im\ (Om))}$	1	1	0	1	1
$r_{Im\ (ßm,\ Om)}$	1	1	0	1	1
$r_{Om\ (Im\ (ßm))}$	1	0	1	1	1

The residue $\hat{r}_{\Omega m(Im)}$ ($\hat{r}_{Im(\Omega m)}$, respectively) is obtained by making reference to M_{Im} ($M_{\Omega m}$, respectively) in $M_{\Omega m}$ (M_{Im}, respectively). Figure 2 marks out the simplification of f_I (f_Ω , respectively), as we have shown in figure 1. Each of these residues allows the localization of f_I and f_Ω between themselves: the simplification of f_I or f_Ω which results from the composition of two local models allows the localization of all faults, fact which is not possible but by the help of the local residues. On the contrary, the resistant couple influences these two first residues. Figure 5 shows that the responses of $\hat{r}_{\Omega m(Im)}$ and $\hat{r}_{Im(\Omega m)}$ afferent to the unobserved exogenous lap inputs are true to the table of theoretical values. (Table 4).

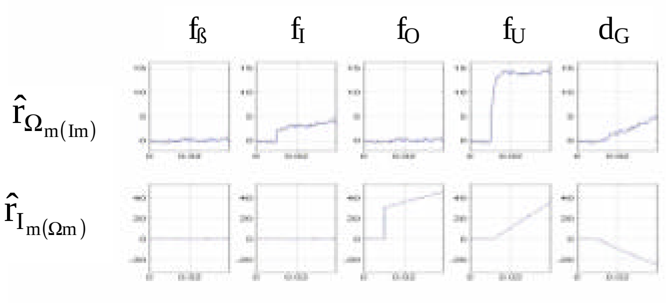

Figure 5. Checking and simulation of the theoretical values for $\hat{r}_{\Omega m(Im)}$ and $\hat{r}_{Im(\Omega m)}$

The association of $M_{ßm}$ with M_{Im} does not simplify the algorithm calculation (any would be the association order). The two residues obtained in this way, $\hat{r}_{ßm(Im)}$ and $\hat{r}_{Im(ßm)}$, do not offer more information about the localization than the local residues. These two residues are sensitive to all faults and insensitive to the couple perturbation: they satisfy the detection objectives.

The last three rows of table 4 refer to three residues, for their calculation three models being necessary and, consequently, all the equations of the local models. The first one is homogenous with ß, the second one with I and the third with O; a comparison with the global residues is required. Consequently, it is possible to find out similar theoretical values. Hence, if the influence of the unobserved exogenous inputs are identical for $\hat{r}_{\beta m(Im(\Omega m))}$ and $r_{\beta m}{}^{*}$ on one hand and $\hat{r}_{Im(\beta m,\Omega m)}$ and $r_{Im}{}^{*}$ on the other hand, for $\hat{r}_{\Omega m(Im(\beta m))}$ and $r_{\Omega m}{}^{*}$ they will be not identical anymore. The difference between the models that allow the calculation of these two last residues is due to the absence of the loop in the composition of the local models and to the presence of the loop in the global model of the system. Thus, the known inputs of the model $M_{\Omega m(Im(\beta m))}$ are I_{ref} , I_m, O_m while the only known input of the global model is I_{ref}.

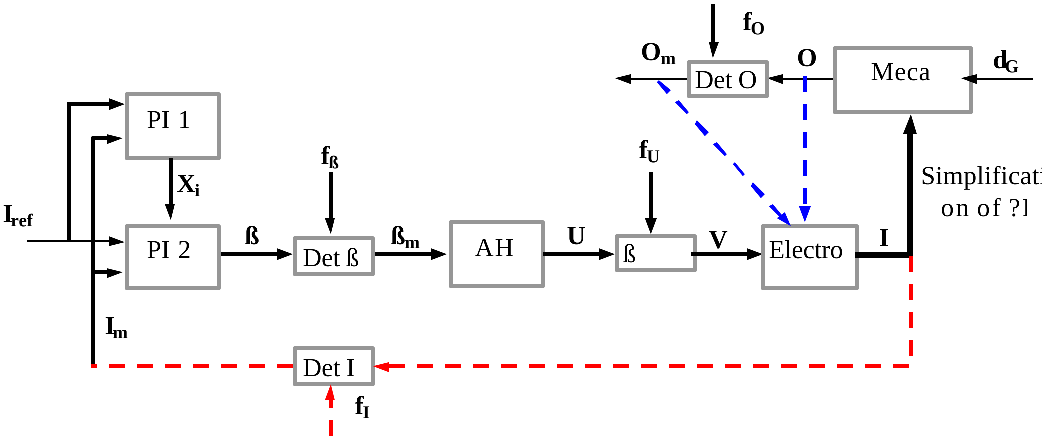

Figure 6. $M_{Om\ (Im\ (ßm))}$: the difference from the global model (dotted lines)

Figure 6 offers a detailed study of the differences between $M_{\Omega m(Im(\beta m))}$ and the global model. The dotted lines mark out the connections that are absent in $M_{\Omega m(Im(\beta m))}$ and present in the global model (the significance of the dotted lines is opposite to that). While $f_ß$, f_O, f_U and d_I are present in both models, f_I depends on the current loop that appear only at the global model. Thus, the presence or the absence of some loops allows the interpretation of the sensibility difference between the residue obtained through the composition of all local models and its homogenous global residue.

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